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THE PRINCIPLE AND APPLICATIONS OF MULTI-PLANE SEPARATION FOR BALANCING MACHINES

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Two-plane separation is a conventional technique of balancing machines for rigid rotors such that each sensor measures the separated effects of equivalent imbalances in two planes. However, some complex rigid rotors such as multicylinder crankshafts need to be balanced by multi-plane correction for reducing mass concentration at two planes. This study verifies the principle of plane separation by using an exact-point influence coefficient approach. From the analysis a generalized algorithm of multiplane separation can be developed. Thus, an unlimited technique of plane separation is provided to improve balancing machines for complex rotors which have several planes in need of correction.

1. INTRODUCTION

In the operation of balancing machines, the principles of plane separation are utilized to separate imbalance effects into discrete planes. By using the cradle-balancing machine, a rotor is placed on two springs and can be fulcrumed at the locations of two chosen balance planes, where the correction masses will be added. When the rotor is driven at a constant speed, imbalance vibrations are due to one of the planes, since the imbalance in another plane which is fulcrumed has no moment about the fulcrumed point. However, the structure of these machines is very complex and difficult to manufacture, and the balancing procedures are also tedious.

The modern balancing machine accomplishes this plane separation of imbalance by means of electrical networks. The rotor is supported by two bearings and vibration sensors at these two points convert response to voltage. Due to the opposite direction of conduction, the voltages and, by means of a voltage amplifier or divider, the imbalance effect due to that of the plane can be reduced to zero at another sensor. By similar reasoning, one of the sensors is readout, which is unaffected by the other plane, to indicate imbalance in this plane.

Some of the earliest general reference work to include discussions of balancing machines were those by Jeffcott [1], Timoshenko [2], and Kroon [3]. Den Hartog [4] described two-plane separation utilized by a cradle-balancing machine for rigid rotors. Similar discussions were presented in the Shock and Vibration Handbook [5], in which several machines and methods for balancing rigid rotors are described. Also, the International Standards Organization (ISO) has issued documentation on balancing machines and plane separation of two-plane rotors [6].

Thus far, plane separation has been discussed in the above mentioned articles for two-plane and rigid rotors only. Some rotors, such as crankshafts, long turbine rotors, and generator rotors, etc., need balancing in several planes for reducing mass concentration

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at two planes. However, the commonly used balancing machines have some difficulty in balancing these complex rotors by using multi-plane correction.

A two-plane, two-sensor, single-speed, exact-point influence coefficient method was developed by Thearle [7] for balancing rigid rotors in the field. This exact-point approach is generally valid and utilizable for balancing the rigid rotor on a balancing machine. Den Hartog [4] also described how the influence coefficient approach was used in Gisholt–Westinghouse balancing machines. Baker [8] provided an influence coefficient method for crankshaft balancing machines.

Actually, the plane-separation and influence coefficient approaches are equivalent and similar, both being based on the linear theory of sensitivity. Rao [9] gave both descriptions about the plane separation and influence coefficient approaches individually for balancing the rigid rotors. Other important reports on investigations involving balancing of rigid rotors were surveyed by Darlow [10]. Until now, studies on plane separation for balancing multi-plane rotors have not been presented.

Kang *et al*. [11] derived formulations of influence coefficient matrices from motion equations for rotors. On the basis of their study, the influence coefficient approach can be verified by an analytical viewpoint and proven that it is not an art but a science. Thus, this study formulates an algorithm of plane separation based on the exact-point influence coefficient approach. From the analysis, a generalized procedure for multiplane separation for balancing a rigid rotor is provided by an inference from two-plane separation and then three-plane separation. This process of multiplane separation can be utilized by a balancing machine to correct a large number of planes simultaneously or successively.

2. PRINCIPLE OF PLANE SEPARATION

2.1. TWO-PLANE SEPARATION

An arbitrary and continuous imbalance distribution in a rigid rotor can be specified in terms of the equivalent components located at two balancing planes in the rotor. Since balancing machines accomplish plane separation of imbalance by means of an electrical network, sensors at two measurement points convert the responses to voltages. By means of a voltage amplifier or divider, the voltage of each imbalance in both planes can be reduced to a value which is equal and opposite to the other. By using the exact-point influence coefficient method, the voltage signals, measured by sensors, can be expressed by

$$\begin{bmatrix} r_a \\ r_b \end{bmatrix} = \begin{bmatrix} \alpha_{aL} & \alpha_{aR} \\ \alpha_{bL} & \alpha_{bR} \end{bmatrix} \begin{bmatrix} u_L \\ u_R \end{bmatrix} = [\boldsymbol{\alpha}]\{\mathbf{u}\}, \qquad (1)$$

where α_{ij} is an influence coefficient to relate the response measured from sensor i to the imbalance on plane j. As shown in Figure 1(a), when a trial mass imbalance, T_L, is placed in the left plane, imbalance responses due to T_L, are measured by sensors as

$$V_{aL} = K_\alpha \alpha_{aL} T_L, \qquad V_{bL} = K_\alpha \alpha_{bL} T_L. \qquad (2a, b)$$

The proportional factor of the left amplifier/divider is calibrated in order to cancel the imbalance effect of T_L on the right plane measurement. In a similar manner, a trial mass imbalance, T_R, is utilized to determine the proportional factor of the right amplifier/divider, as shown in Figure 1(b). These factors are calibrated as

$$K_L = \alpha_{bL}/\alpha_{aL}, \qquad K_R = \alpha_{aR}/\alpha_{bR}. \qquad (3a, b)$$

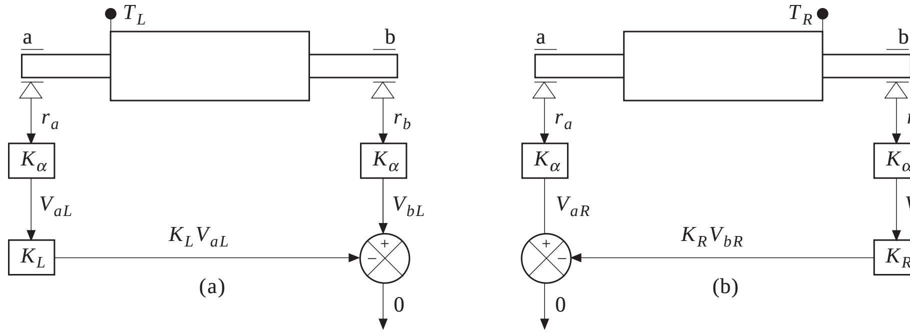

Figure 1. Calibration of plane separation with trial operations (a) left plane separation, (b) right plane separation.

Using these two factors, a plane separation of two balancing planes can be attained. The final outputs, as shown in Figure 2, are obtained as

$$R_a = K_\alpha(\alpha_{aL} - K_R\alpha_{bL})U_L, \qquad R_b = K_\alpha(\alpha_{bR} - K_L\alpha_{aR})U_R \tag{4a, b}$$

The above analysis shows that the left sensor and the right sensor read out only the equivalent effects in the left plane and the right plane, respectively.

2.2. THREE-PLANE SEPARATION

When a complex rotor is balanced by placing correction masses on three planes, three sensors located at points a, b, and c are used to measure imbalance responses. The relationships between imbalances and responses can be represented by exact-point influence coefficients as follows:

$$\begin{bmatrix} r_a \\ r_b \\ r_c \end{bmatrix} = \begin{bmatrix} \alpha_{aa} & \alpha_{ab} & \alpha_{ac} \\ \alpha_{ba} & \alpha_{bb} & \alpha_{bc} \\ \alpha_{ca} & \alpha_{cb} & \alpha_{cc} \end{bmatrix} \begin{bmatrix} U_a \\ U_b \\ U_c \end{bmatrix} \tag{5}$$

The usage of amplifiers/dividers, as shown in Figure 3, can accomplish the separation of planes for this rotor. For plane a as an example, the imbalance effects due to the second and the third planes can be deleted from the measurement of sensor a by the following

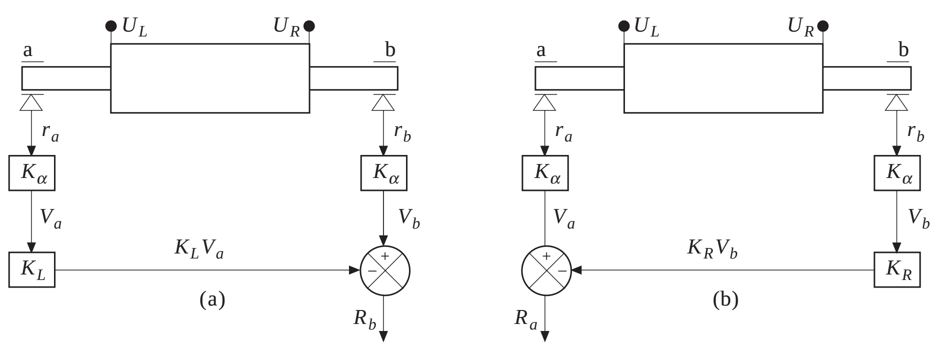

Figure 2. Imbalance readouts due to two-plane separation (a) measurement for the right plane, (b) measurement for the left plane.

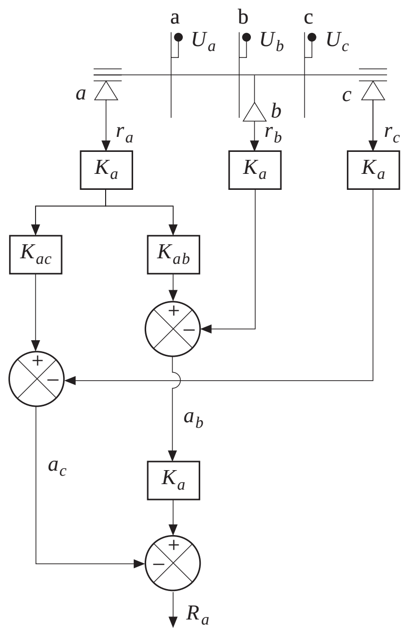

Figure 3. Three-plane separation.

two steps. In the first step a trial mass imbalance, T_2, is placed on the second plane. The proportional factors, K_{ab}, and K_{ac}, of the voltage dividers are calibrated to be

$$K_{ab} = \alpha_{bb}/\alpha_{ab}, \qquad K_{ac} = \alpha_{cb}/\alpha_{ab} \tag{6a, b}$$

in order to output at a_c, a_b being zero. In the second step a trial mass imbalance, T_3, is placed on the third plane. The proportional factor K_a is calibrated so that the result from the final output of sensor a is zero, i.e., $R_a = 0$. Thus,

$$K_a = (K_{ac}\alpha_{ac} - \alpha_{cc})/(K_{ab}\alpha_{ac} - \alpha_{bc}) \tag{7}$$

With these calibrated factors as shown by equations (6a), (6b) and (7), the final output of the imbalance response measured by sensor a is determined by

$$\begin{aligned} R_a &= K_\alpha[K_a(K_{ab}r_a - r_b) - (K_{ac}r_a - r_c)] \\ &= K_\alpha U_a \frac{(\alpha_{cb}\alpha_{ac} - \alpha_{cc}\alpha_{ab})(\alpha_{bb}\alpha_{aa} - \alpha_{ab}\alpha_{ba}) + (\alpha_{ca}\alpha_{ab} - \alpha_{cb}\alpha_{aa})(\alpha_{bb}\alpha_{ac} - \alpha_{bc}\alpha_{ab})}{\alpha_{ab}(\alpha_{bb}\alpha_{ac} - \alpha_{bc}\alpha_{ab})} \\ &= K_A U_a \end{aligned} \tag{8}$$

In the first layer the proportional factors K_{ab} and K_{ac} are used to delete an imbalance response due to U_2 measured by substracting signals measured from sensors b and c. In the second layer a proportional factor K_a is used to delete the residual responses due to imbalance, U_3, on the third plane in signals a_c and a_b by subtraction. Thus, the first and the second layers are used to cancel U_b and U_c effects, respectively. In a similar manner the second plane and the third plane can be separated into the forms

$$R_b = K_B U_b, \qquad R_c = K_C U_c \tag{9a, b}$$

2.3. MULTI-PLANE SEPARATION

Figure 4 shows that a rotor has N balancing planes which are required to place correction masses. First, N sensors are installed to measure imbalance responses. Using a trial mass imbalance, T_2, placed on the second plane and readouts $r_{21}, r_{31}, \ldots, r_{N1}$ calibrated to zero, the proportional factors, K_{21}, K_{31}, and K_{N1}, of N-1 amplifiers/dividers are obtained. In a succeeding process trial masses, $T_3, T_4, \ldots, T_N$, are placed on the third, fourth, ... , and Nth planes, respectively, to calibrate $K_{32}, K_{42}, \ldots, K_{N2}$ in the second layer, $K_{43}, K_{53}, \ldots, K_{N3}$ in the third layer, . . . , and $K_{N,N-1}$ in the (N-1)th layer.

After these procedures are accomplished, the effects of imbalances $U_2, U_3, \ldots$ and U_N on the second, the third, . . . and the Nth planes, respectively, cannot be presented in the final readout, R_1, from the first sensor. Similarly, other N-1 planes are separated from each other. Final readouts $R_2, R_3, \ldots$, and R_N contain an imbalance effect due only to the corresponding plane.

3. THE APPLICATION OF PLANE SEPARATION

For balancing complex rotors which have a large number of planes needing correction with a balancing machine, one may use the same-number plane separation or three-plane separation. In three-plane separation three sensors must be utilized to measure imbalance responses; and imbalances in these planes are corrected successively, whereby two sensors

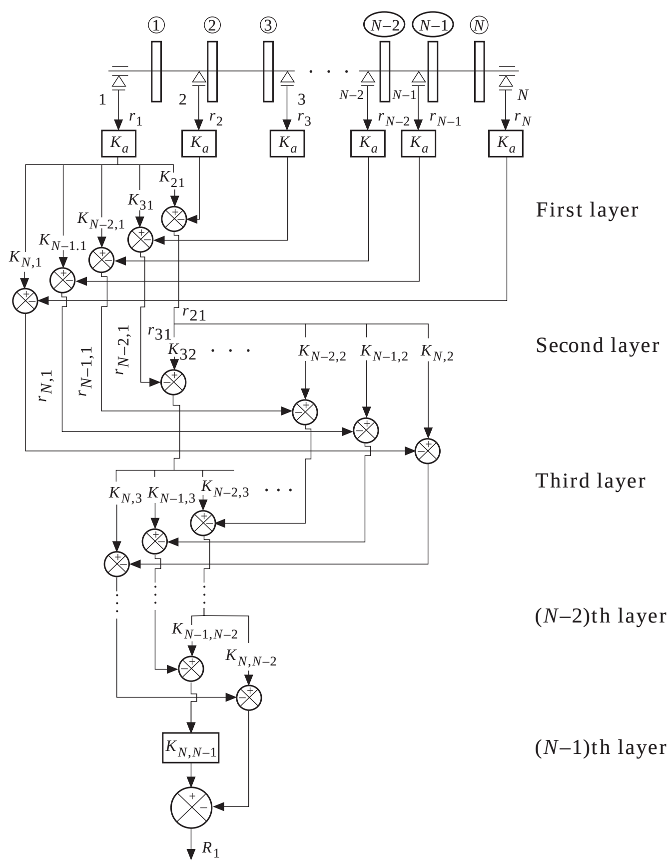

Figure 4. N-plane separation.

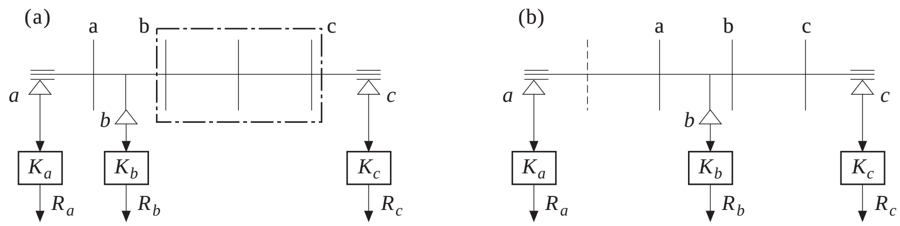

Figure 5. A rotor with four planes, balanced by three-plane separation (a) at the first step, (b) at the second step.

are generally located at both ends of the rotors and one sensor is movable. For example, a rotor having four planes is shown in Figure 5. By using three-plane separation, the three sensors measure imbalance effects in the corresponding plane individually. The first left plane is separated in the first step and corrected from the determination of equivalent imbalances. In the second step, the three right planes are calibrated to separate from each other, and the imbalance effects are measured from the corresponding sensors. With these two steps, imbalances in the four planes can be determined and corrected.

The figures in which a dashed line is used to illustrate the disk have no imbalance effects because the balancing correction and planes included in the chain loops are combined equivalently to a two-plane rotor.

The application of three-plane separation for more than five planes is a successive process, as shown in Figure 6, in a step to balance one plane which is separated from the other planes.

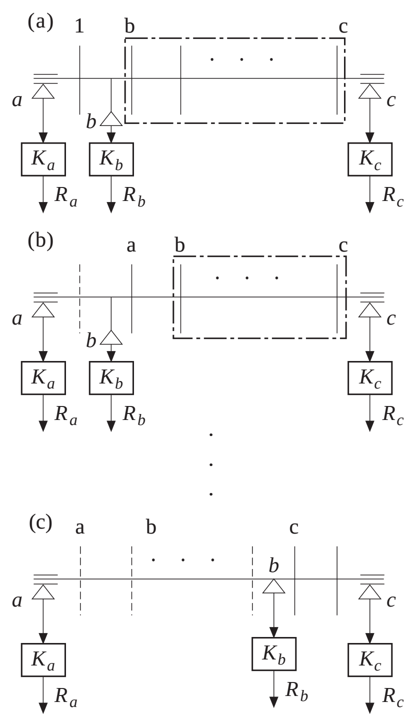

Figure 6. A rotor with N planes, balanced by three-plane separation (a) at the first step, (b) at the second step, (c) at the last step.

4. CONCLUSION

The influence coefficient approach determines all imbalances simultaneously, whilst the plane separation approach determines imbalances successively within each plane. Apparent advantages and disadvantages may be associated with each approach due to different applications of balancing machines for various types of rotors.

This study has presented a generalized technique for plane separation specifically developed for balancing machines. This technique has been extended from two-plane separation to multiplane separation and demonstrated by using the theory of exact-point influence coefficients. It has a sound theoretical foundation and unlimited usage. On the basis of this analysis, three-plane and N-plane separations may be applied to balance rotors with an arbitrary number of planes in successive and in simultaneous manners.

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APPENDIX A: NOMENCLATURE

f, F imbalance force
K proportional factor of amplifier/divider
M imbalance couple
r measured response
R final readout
U imbalance (the product of rotor mass and eccentricity)
V voltage transformed by sensor
T imbalance of trial mass
α influence coefficient